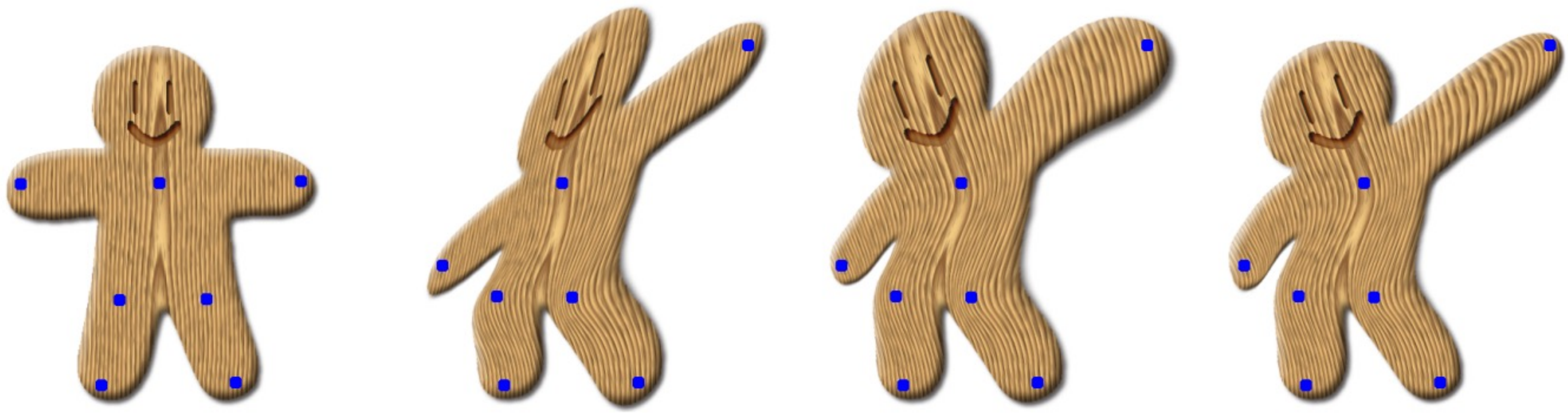


Global and Local Image Warping

Jun-Yan Zhu

16-726 Learning-based Image Synthesis, Spring 2023

Logistics

1. Homework 0 released

Assignment #0 - How to submit assignments?

Released on Monday 01/23/2023

Due Date: Monday 01/30/2023 23:59
Download:

Late Policy

- You have free 5 late days.
- You can use late days for assignments. A late day extends the deadline 24 hours.
- Once you have used all 5 late days, the penalty is 10% for each additional late day.

How to submit assignments? (in general)

For each assignment, you will be required to submit two packages unless specified otherwise: your **code** and your **webpage**:

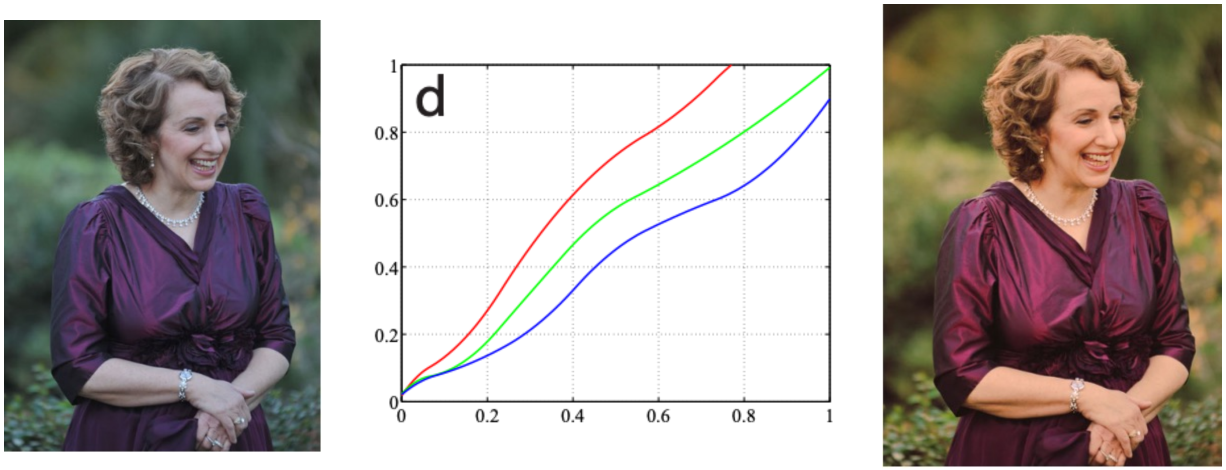
Submit code to Canvas

For every assignment you should create a `main.py` that can be used to run all your code for the assignment, and a `README.md` file that contains all required documentation. Place all source code used to generate your results, as well as any documentation required to use the code, in a folder named `andrewid_code_projX` where X is the hw number. Zip the whole folder and submit the zip to Canvas. Here is an example of your folder structure:

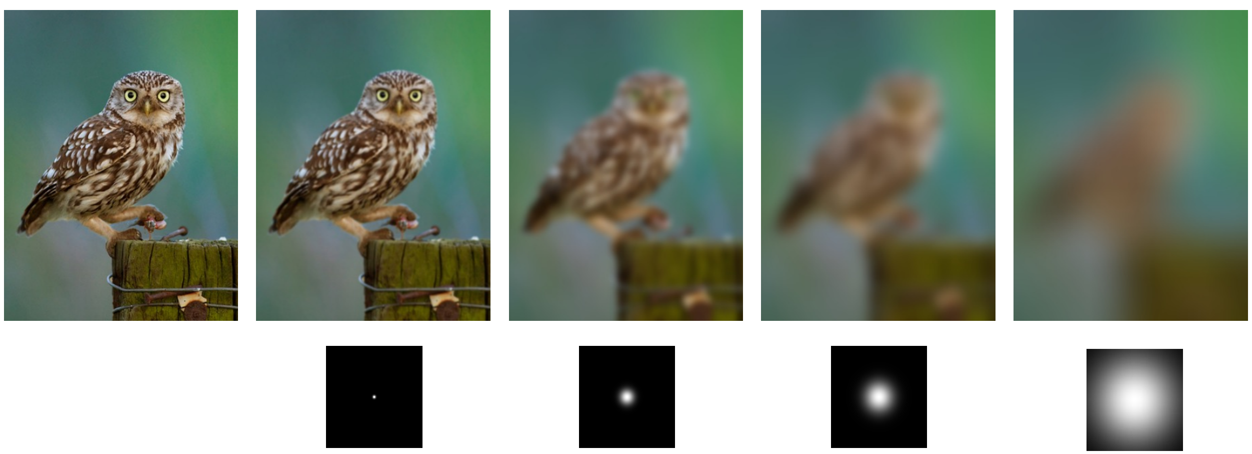
```
zhiqiul_code_proj1/
    main.py
    README.md
    utils.py
    ....
## zip the whold folder to zhiqiul_code_proj1.zip:
```

Review: how to create an image

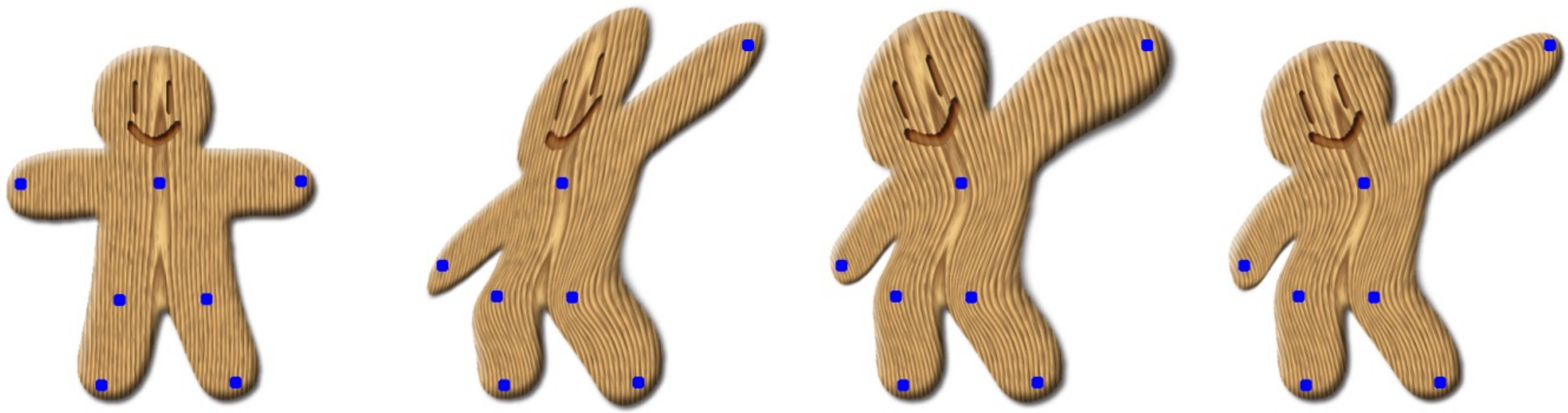
- Pointwise Processing:



- Image Filtering



σ = 1 pixel σ = 5 pixels σ = 10 pixels σ = 30 pixels



Global and Local Image Warping

Jun-Yan Zhu

16-726 Learning-based Image Synthesis, Spring 2023

Image Transformations

image filtering: change ***range*** of image

$$g(x) = T(f(x))$$

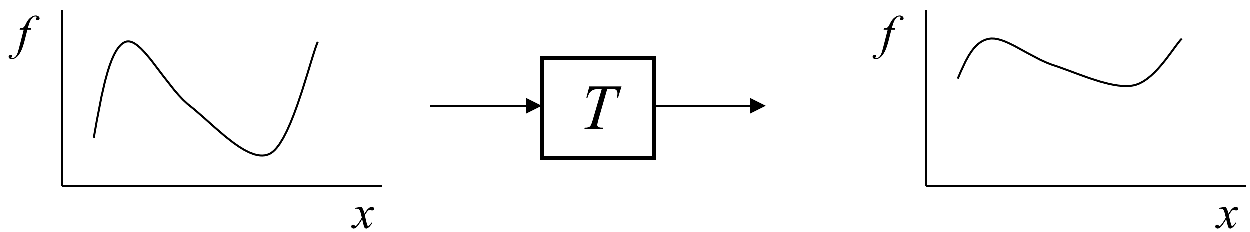
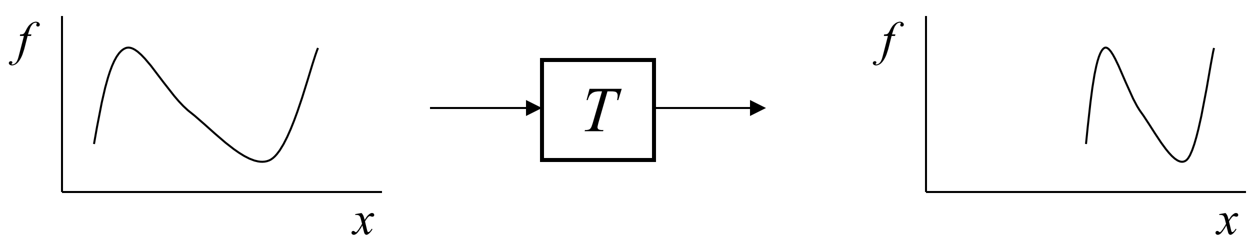


image warping: change ***domain*** of image

$$g(x) = f(T(x))$$



Many slides are adopted from Alyosha Efros and Steve Seitz, among other great people.
See book chapter (Szeliski Sec 2.1.2)

Image Transformations

image filtering: change ***range*** of image

$$g(x) = T(f(x))$$

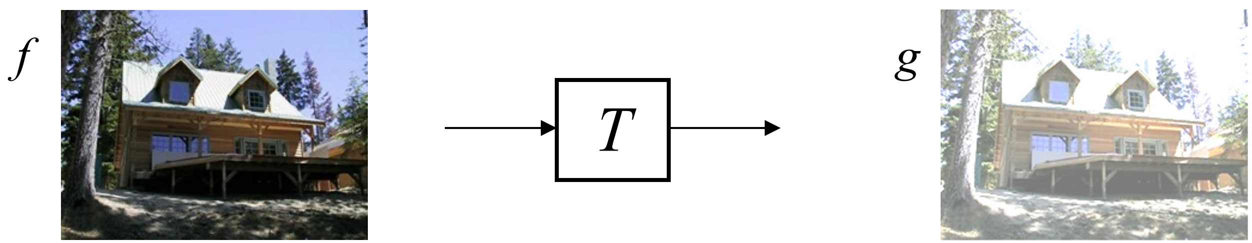
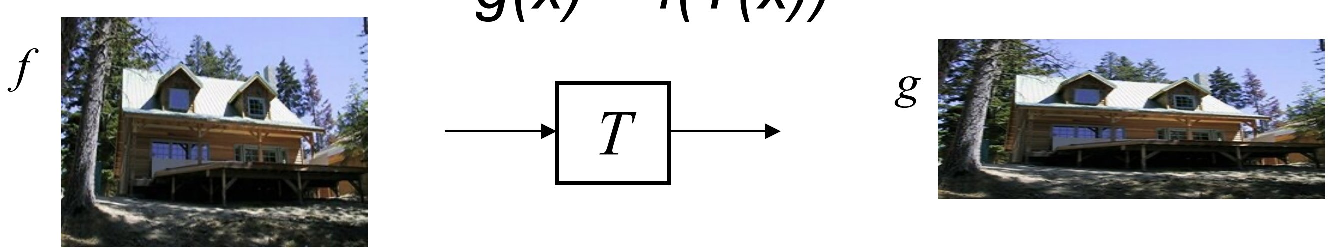


image warping: change ***domain*** of image

$$g(x) = f(T(x))$$



Parametric (global) warping

Examples of parametric warps:

translation

rotation

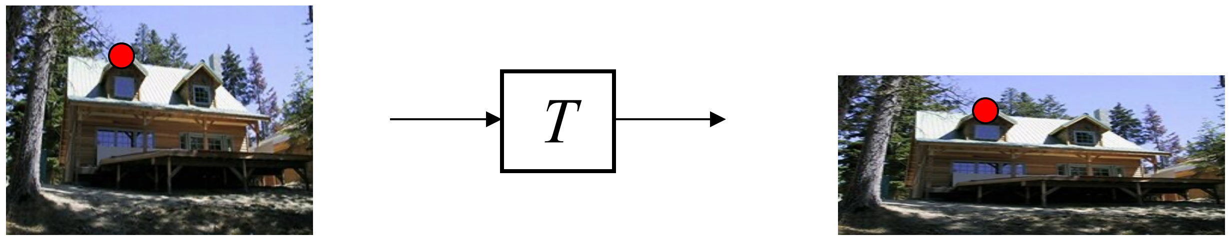
aspect

affine

perspective

cylindrical

Parametric (global) warping



$\mathbf{p} = (x,y)$ $\mathbf{p'} = (x',y')$

Transformation T is a coordinate-changing machine:

$$p' = T(p)$$

What does it mean that *T* is global?

- Is the same for any point p
- can be described by just a few numbers (parameters)

Let's represent a linear *T* as a matrix:

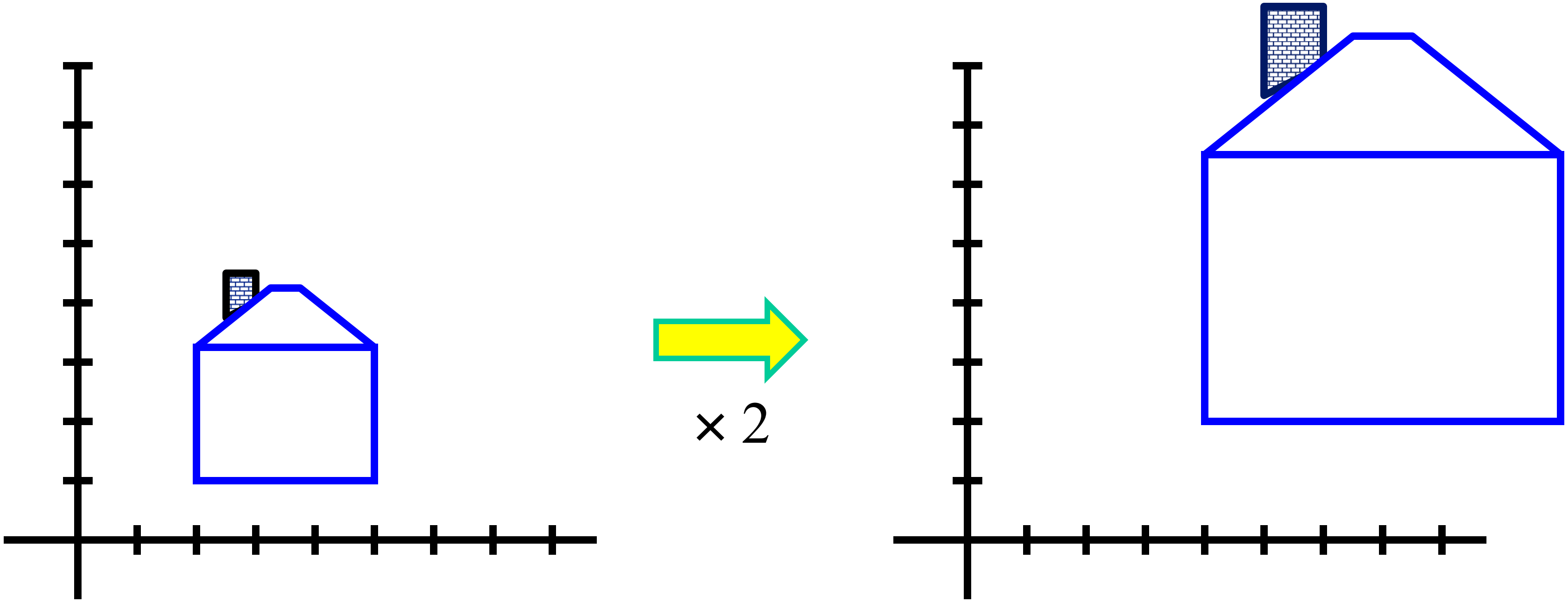
$$p' = \mathbf{M}p$$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \mathbf{M} \begin{bmatrix} x \\ y \end{bmatrix}$$

Scaling

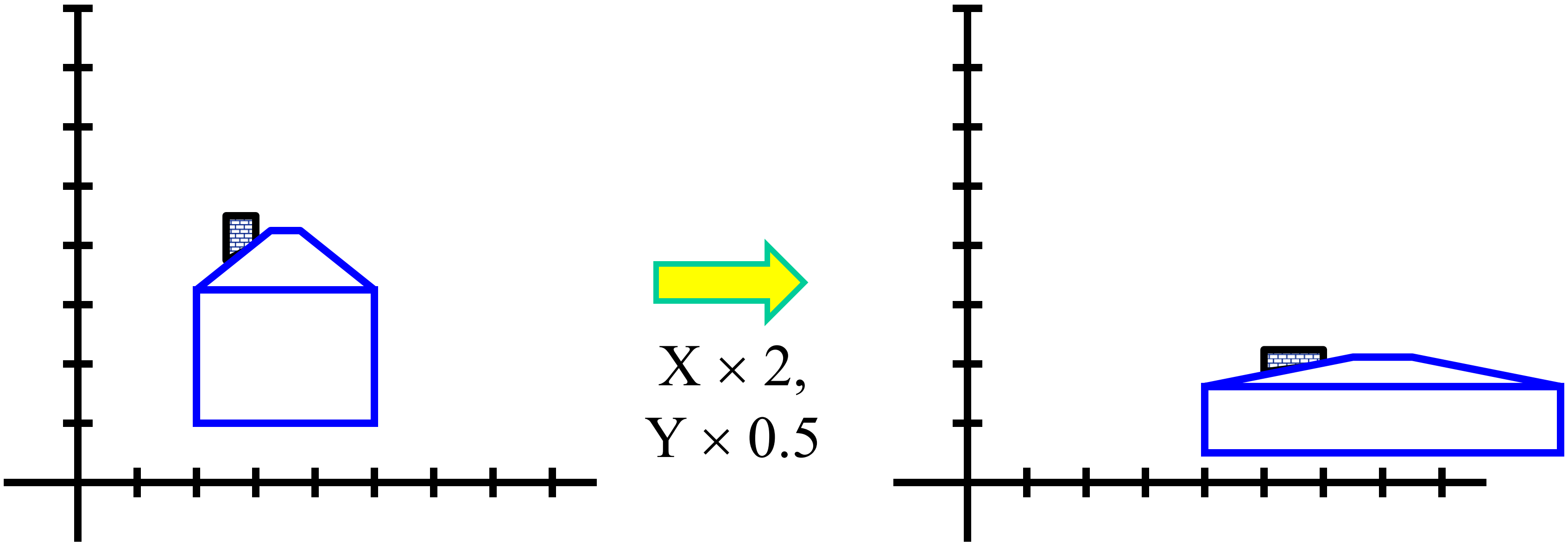
Scaling a coordinate means multiplying each of its components by a scalar

Uniform scaling means this scalar is the same for all components:



Scaling

Non-uniform scaling: different scalars per component:



Scaling

Scaling operation:

$$x' = ax$$

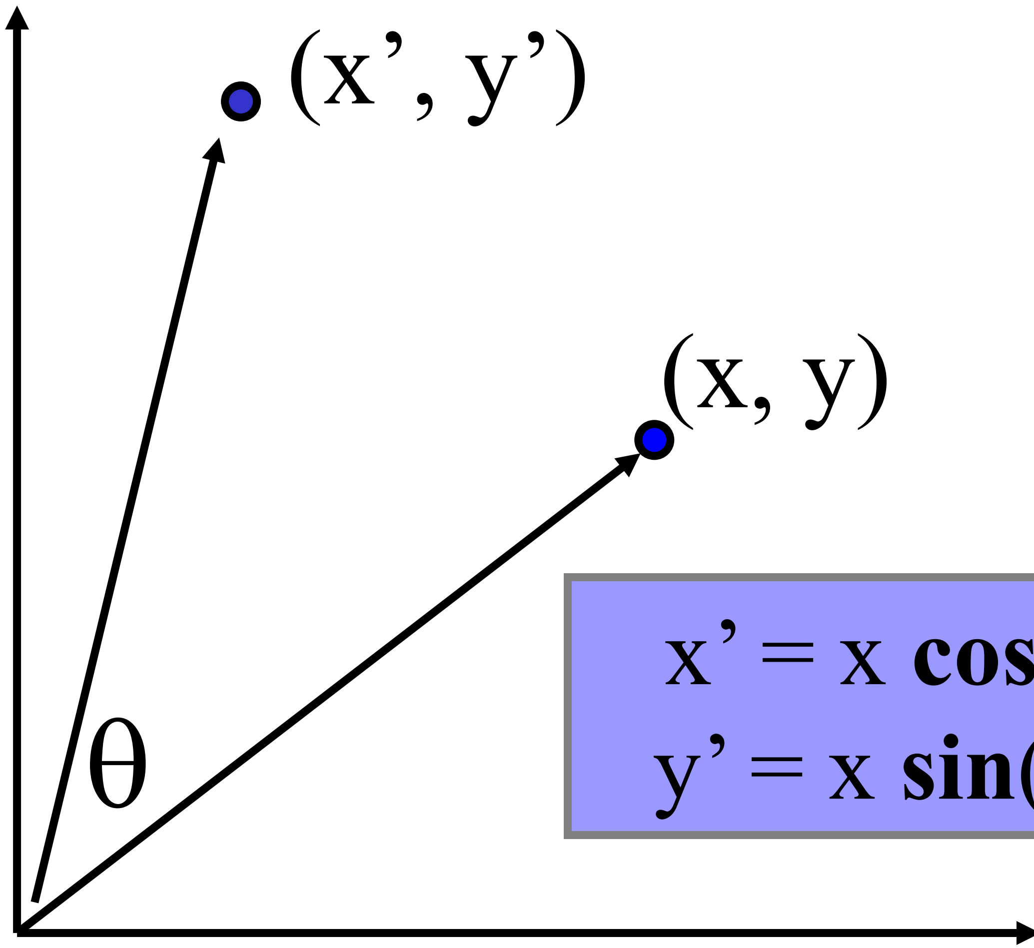
$$y' = by$$

Or, in matrix form:

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \underbrace{\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}}_{\textit{scaling matrix } S} \begin{bmatrix} x \\ y \end{bmatrix}$$

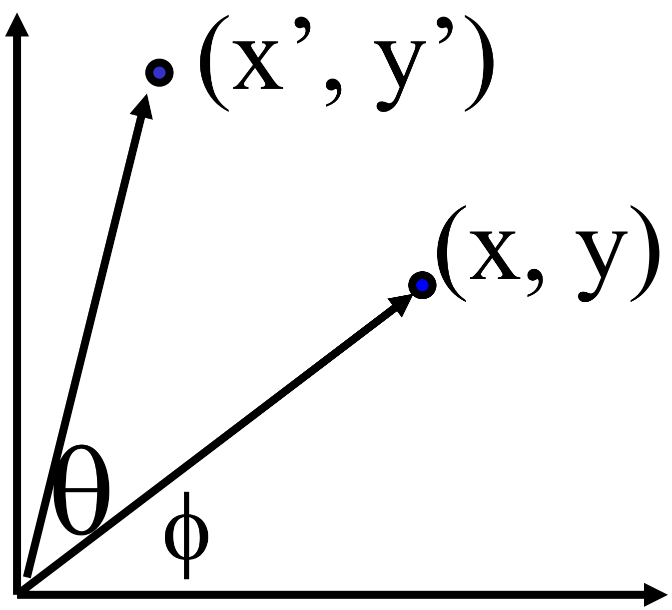
What's inverse of S?

2-D Rotation



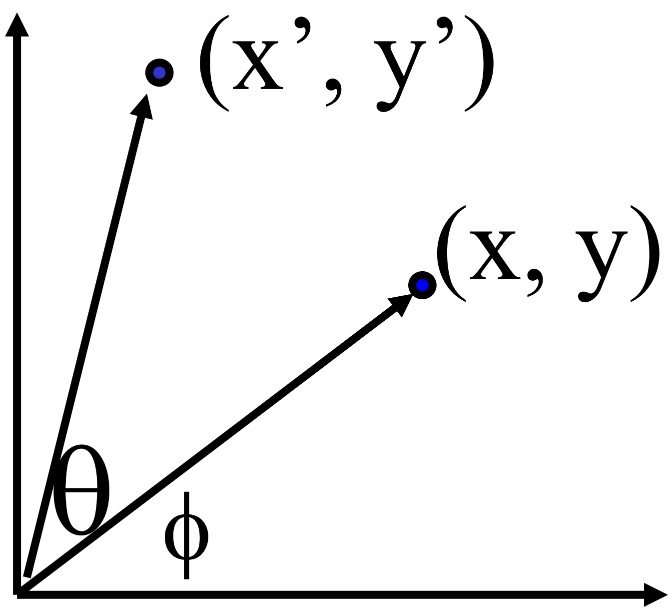
$$x' = x\ \mathbf{cos}(\theta)\ \text{-}\ y\ \mathbf{sin}(\theta)$$
$$y' = x\ \mathbf{sin}(\theta) + y\ \mathbf{cos}(\theta)$$

2-D Rotation



$x = r \cos(\phi)$
$y = r \sin(\phi)$
$x' = r \cos(\phi + \theta)$
$y' = r \sin(\phi + \theta)$

2-D Rotation



$x = r \cos(\phi)$

$y = r \sin(\phi)$

$x' = r \cos(\phi + \theta)$

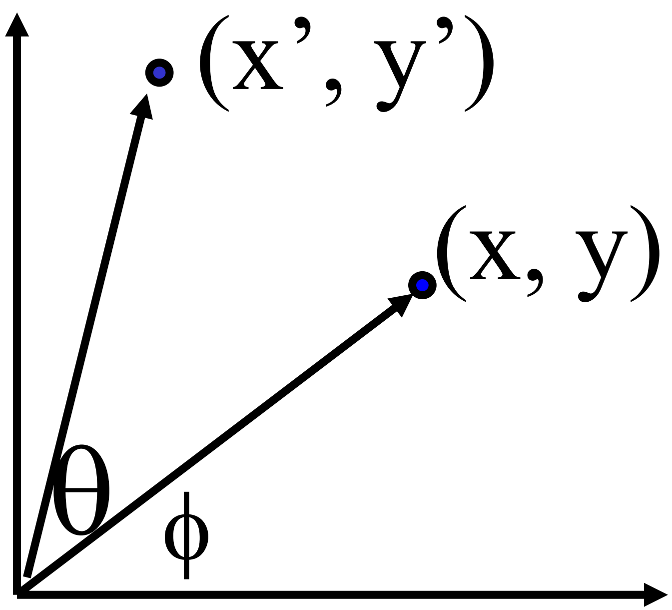
$y' = r \sin(\phi + \theta)$

Trig Identity…

$x' = r \cos(\phi) \cos(\theta) - r \sin(\phi) \sin(\theta)$

$y' = r \sin(\phi) \cos(\theta) + r \cos(\phi) \sin(\theta)$

2-D Rotation



$x = r \cos(\phi)$

$y = r \sin(\phi)$

$x' = r \cos(\phi + \theta)$

$y' = r \sin(\phi + \theta)$

Trig Identity…

$x' = r \cos(\phi) \cos(\theta) - r \sin(\phi) \sin(\theta)$

$y' = r \sin(\phi) \cos(\theta) + r \cos(\phi) \sin(\theta)$

Substitute…

$x' = x \, \mathbf{cos}(\theta) - y \, \mathbf{sin}(\theta)$

$y' = x \, \mathbf{sin}(\theta) + y \, \mathbf{cos}(\theta)$

2-D Rotation

This is easy to capture in matrix form:

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \underbrace{\begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix}}_{\mathbf{R}} \begin{bmatrix} x \\ y \end{bmatrix}$$

Even though sin(θ) and cos(θ) are nonlinear functions of θ,

- ***x' is a linear combination of x and y***
- ***y' is a linear combination of x and y***

What is the inverse transformation?

- Rotation by –θ
- For rotation matrices $\mathbf{R}^{-1} = \mathbf{R}^T$

2x2 Matrices

What types of transformations can be represented with a 2x2 matrix?

2D Identity?

$$x' = x$$
$$y' = y$$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

2D Scale around (0,0)?

$$\boldsymbol{x}' = \boldsymbol{s}_{x} * \boldsymbol{x}$$
$$\boldsymbol{y}' = \boldsymbol{s}_{y} * \boldsymbol{y}$$

$$\begin{bmatrix} \boldsymbol{x}' \\ \boldsymbol{y}' \end{bmatrix} = \begin{bmatrix} \boldsymbol{s}_{x} & 0 \\ 0 & \boldsymbol{s}_{y} \end{bmatrix} \begin{bmatrix} \boldsymbol{x} \\ \boldsymbol{y} \end{bmatrix}$$

2x2 Matrices

What types of transformations can be represented with a 2x2 matrix?

2D Rotate around (0,0)?

$$x' = \cos\Theta * x - \sin\Theta * y$$
$$y' = \sin\Theta * x + \cos\Theta * y$$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \cos\Theta & -\sin\Theta \\ \sin\Theta & \cos\Theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

2D Shear?

$$x' = x + sh_x * y$$
$$y' = sh_y * x + y$$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 1 & sh_x \\ sh_y & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

2x2 Matrices

What types of transformations can be represented with a 2x2 matrix?

2D Mirror about Y axis?

$$x' = -x$$
$$y' = y$$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

2D Mirror over (0,0)?

$$x' = -x$$
$$y' = -y$$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

2x2 Matrices

What types of transformations can be represented with a 2x2 matrix?

2D Translation?

$$x' = x + t_x$$
$$y' = y + t_y$$

NO!

Only linear 2D transformations can be represented with a 2x2 matrix

All 2D Linear Transformations

Linear transformations are combinations of …

- Scale,
- Rotation,
- Shear, and
- Mirror

Properties of linear transformations:

- Origin maps to origin
- Lines map to lines
- Parallel lines remain parallel
- Ratios are preserved
- Closed under composition

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} e & f \\ g & h \end{bmatrix} \begin{bmatrix} i & j \\ k & l \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

Homogeneous Coordinates

Q: How can we represent translation as a 3x3 matrix?

$$x' = x + t_x$$

$$y' = y + t_y$$

Homogeneous Coordinates

Homogeneous coordinates

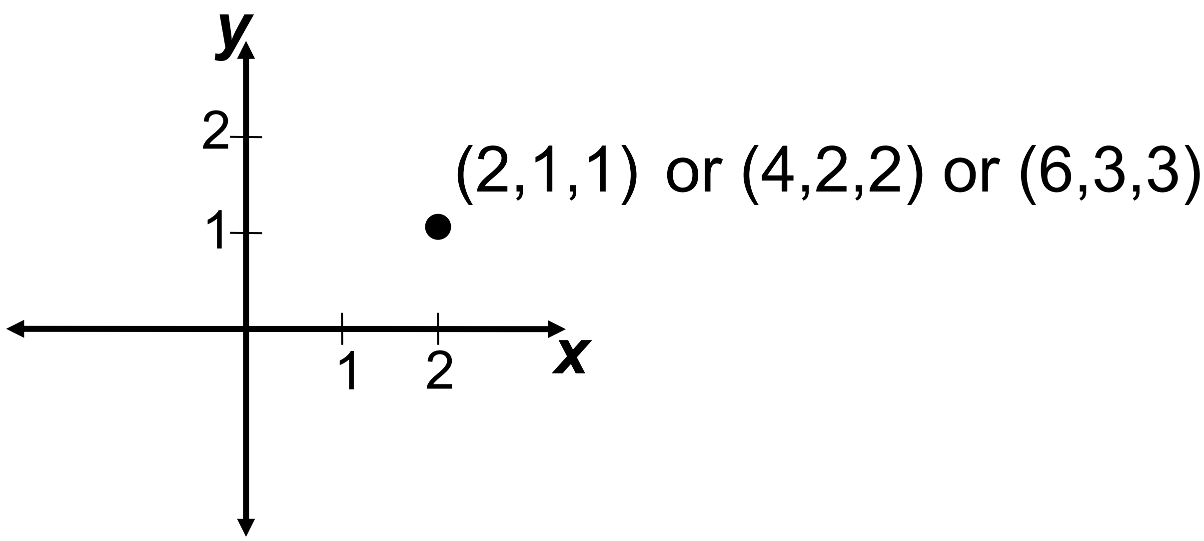
- represent coordinates in 2 dimensions with a 3-vector

$$\begin{bmatrix} x \\ y \end{bmatrix} \xrightarrow{\text{homogeneous coords}} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Homogeneous Coordinates

Add a 3rd coordinate to every 2D point

- (x, y, w) represents a point at location (x/w, y/w)
- (x, y, 0) represents a point at infinity
- (0, 0, 0) is not allowed



Convenient coordinate system to represent many useful transformations

Homogeneous Coordinates

Q: How can we represent translation as a 3x3 matrix?

$$x' = x + t_x$$

$$y' = y + t_y$$

A: Using the rightmost column:

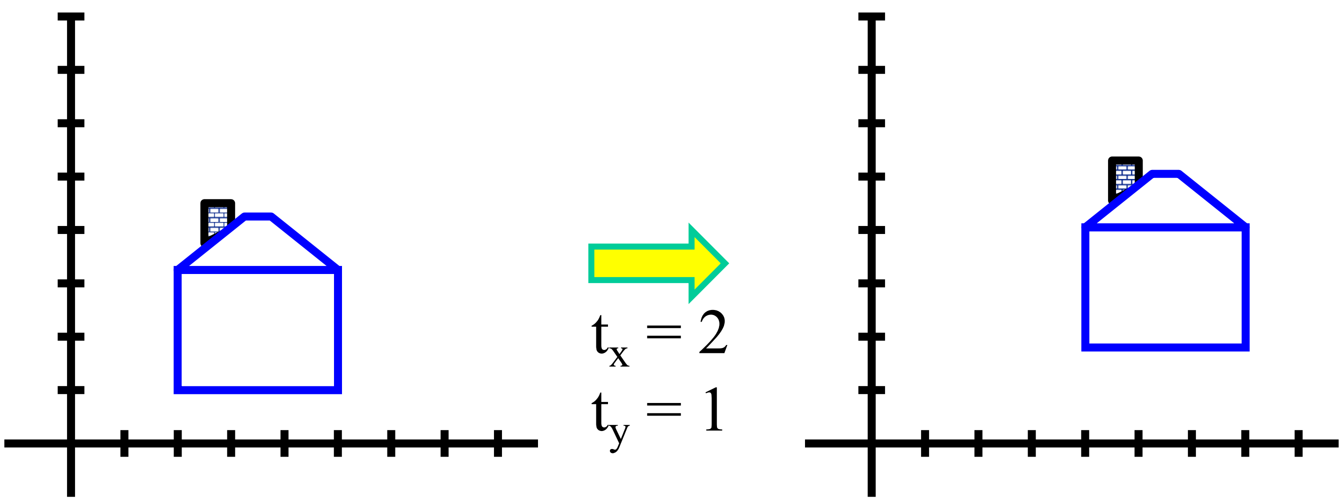
$$\mathbf{T}ranslation = \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix}$$

Translation

Example of translation

Homogeneous Coordinates

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} x + t_x \\ y + t_y \\ 1 \end{bmatrix}$$



Basic 2D Transformations

Basic 2D transformations as 3x3 matrices

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Translate

$$\begin{bmatrix} \boldsymbol{x'} \\ \boldsymbol{y'} \\ 1 \end{bmatrix} = \begin{bmatrix} \boldsymbol{s_x} & 0 & 0 \\ 0 & \boldsymbol{s_y} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \boldsymbol{x} \\ \boldsymbol{y} \\ 1 \end{bmatrix}$$

Scale

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\Theta & -\sin\Theta & 0 \\ \sin\Theta & \cos\Theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Rotate

$$\begin{bmatrix} \boldsymbol{x'} \\ \boldsymbol{y'} \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & \boldsymbol{sh_x} & 0 \\ \boldsymbol{sh_y} & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \boldsymbol{x} \\ \boldsymbol{y} \\ 1 \end{bmatrix}$$

Shear

Matrix Composition

Transformations can be combined by matrix multiplication

$$\begin{bmatrix} x' \\ y' \\ w' \end{bmatrix} = \left(\begin{bmatrix} 1 & 0 & tx \\ 0 & 1 & ty \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\Theta & -\sin\Theta & 0 \\ \sin\Theta & \cos\Theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} sx & 0 & 0 \\ 0 & sy & 0 \\ 0 & 0 & 1 \end{bmatrix} \right) \begin{bmatrix} x \\ y \\ w \end{bmatrix}$$

p' = $T(t_x,t_y)$ $R(\Theta)$ $S(s_x,s_y)$ **p**

Does the order of multiplication matter?

Affine Transformations

$$\begin{bmatrix} x' \\ y' \\ w \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ w \end{bmatrix}$$

Affine transformations are combinations of …

- Linear transformations, and
- Translations

Properties of affine transformations:

- Origin does not necessarily map to origin
- Lines map to lines
- Parallel lines remain parallel
- Ratios are preserved
- Closed under composition
- Models change of basis

Will the last coordinate *w* always be 1?

Projective Transformations

$$\begin{bmatrix} x' \\ y' \\ w' \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \begin{bmatrix} x \\ y \\ w \end{bmatrix}$$

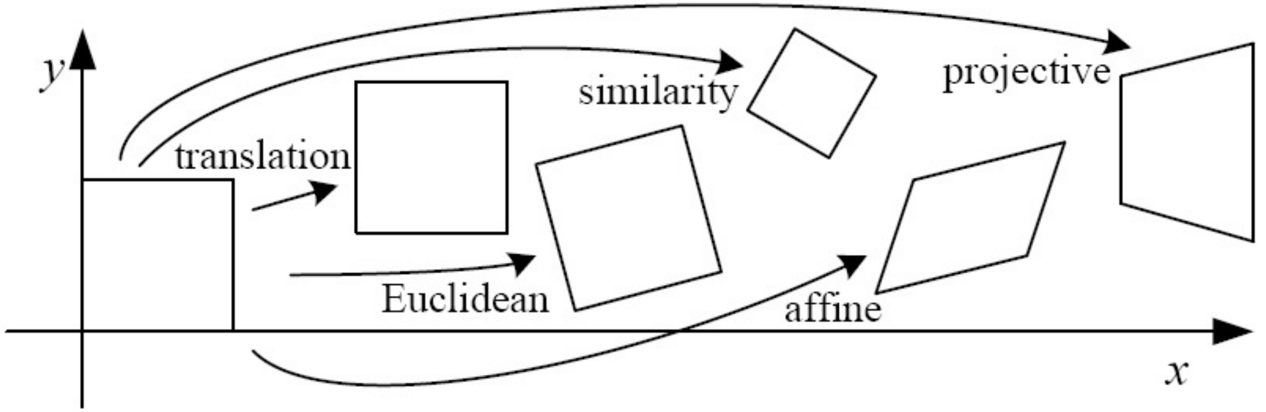
Projective transformations …

- Affine transformations, and
- Projective warps

Properties of projective transformations:

- Origin does not necessarily map to origin
- Lines map to lines
- Parallel lines do not necessarily remain parallel
- Ratios are not preserved
- Closed under composition
- Models change of basis

2D image transformations



Name	Matrix	# D.O.F.	Preserves:	Icon
translation	$\begin{bmatrix} I \mid t \end{bmatrix}_{2\times 3}$			
rigid (Euclidean)	$\begin{bmatrix} R \mid t \end{bmatrix}_{2\times 3}$			
similarity	$\begin{bmatrix} sR \mid t \end{bmatrix}_{2\times 3}$			
affine	$\begin{bmatrix} A \end{bmatrix}_{2\times 3}$			
projective	$\begin{bmatrix} \tilde{H} \end{bmatrix}_{3\times 3}$			

These transformations are a nested set of groups

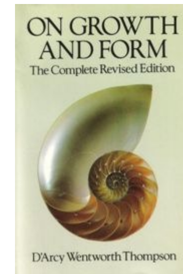
- Closed under composition and inverse is a member

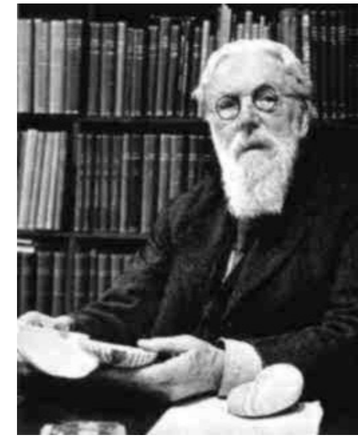
Image Warping in Biology

D'Arcy Thompson

- http://www-groups.dcs.st-and.ac.uk/~history/Miscellaneous/darcy.html
- http://en.wikipedia.org/wiki/D'Arcy_Thompson

Importance of shape and structure in evolution





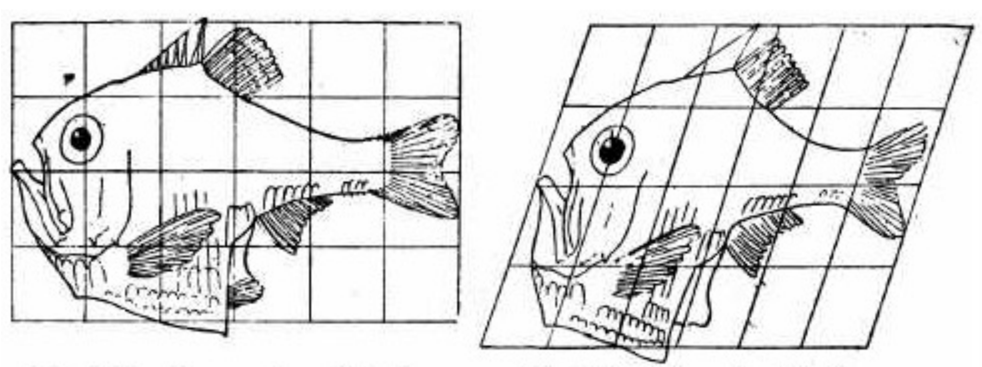
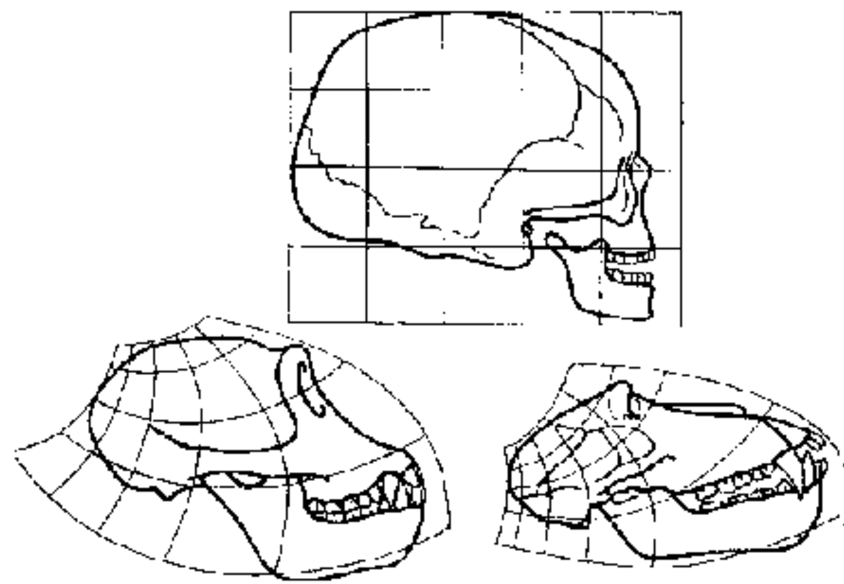


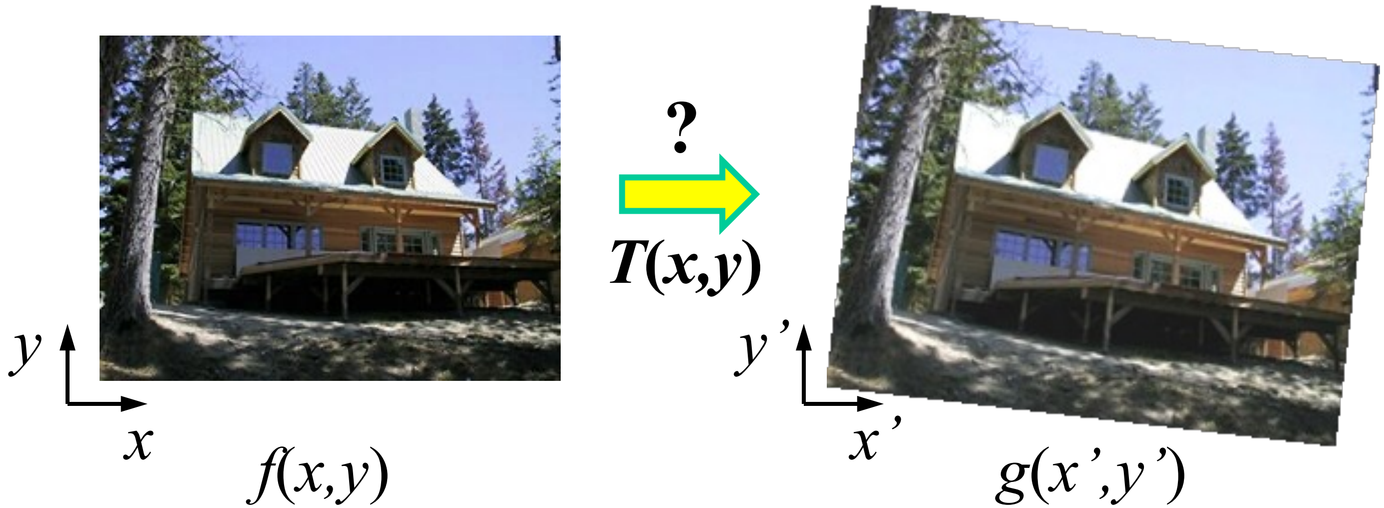
Fig. 517. *Argyropelecus Olfersi.*   Fig. 518. *Sternoptyx diaphana.*



Skulls of a human, a chimpanzee and a baboon and transformations between them

Slide by Durand and Freeman

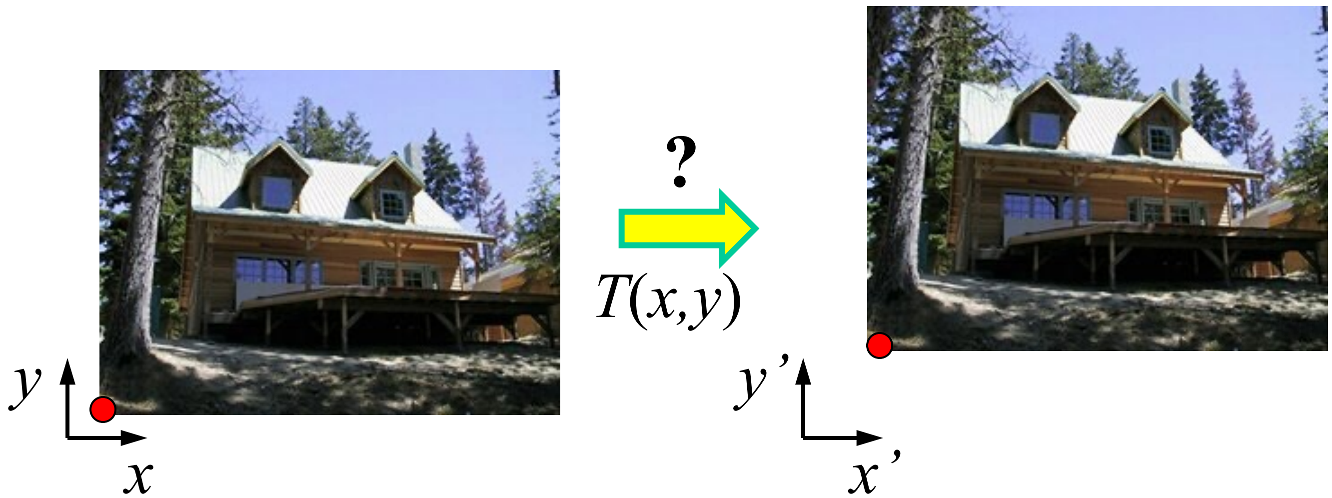
Recovering Transformations



What if we know *f* and *g* and want to recover the transform T?

- e.g. better align images from Project 1
- willing to let user provide correspondences
  - How many do we need?

Translation: # correspondences?



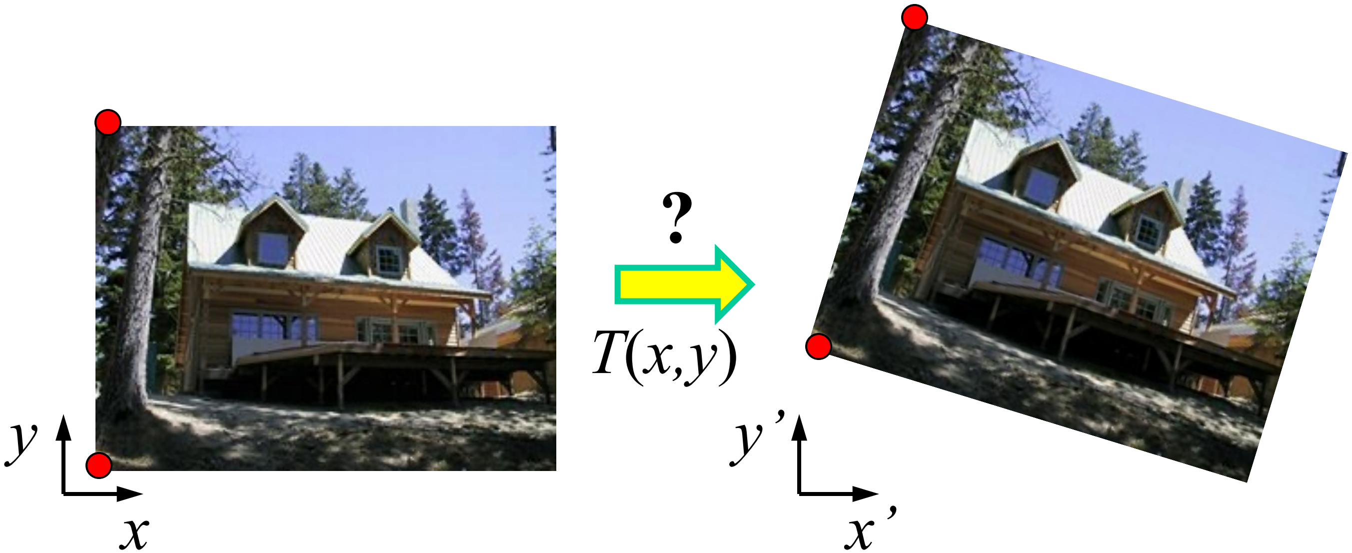
How many correspondences needed for translation?

How many Degrees of Freedom?

What is the transformation matrix?

$$\mathbf{M} = \begin{bmatrix} 1 & 0 & p'_x - p_x \\ 0 & 1 & p'_y - p_y \\ 0 & 0 & 1 \end{bmatrix}$$

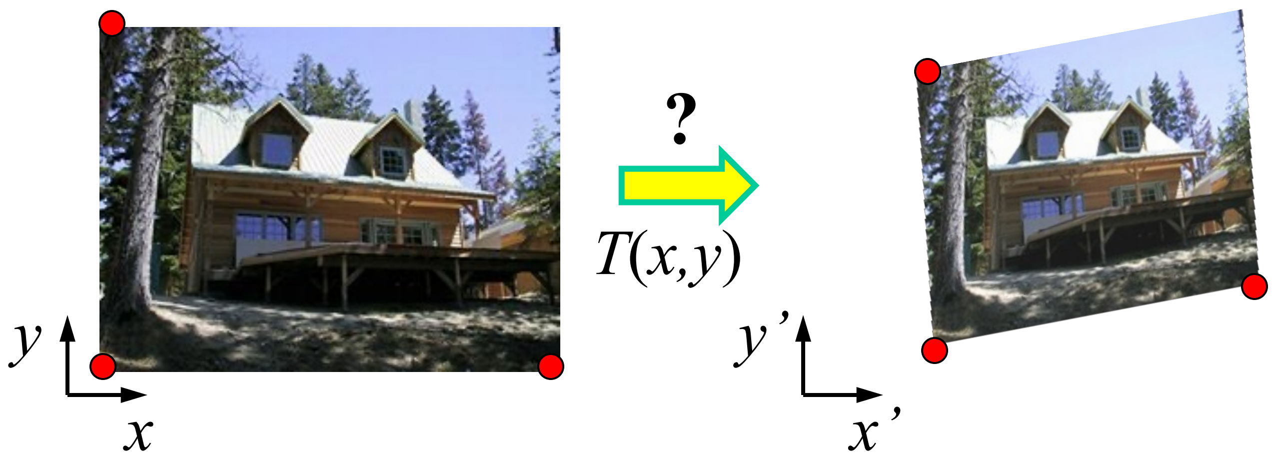
Euclidian: # correspondences?



How many correspondences needed for translation+rotation?

How many DOF?

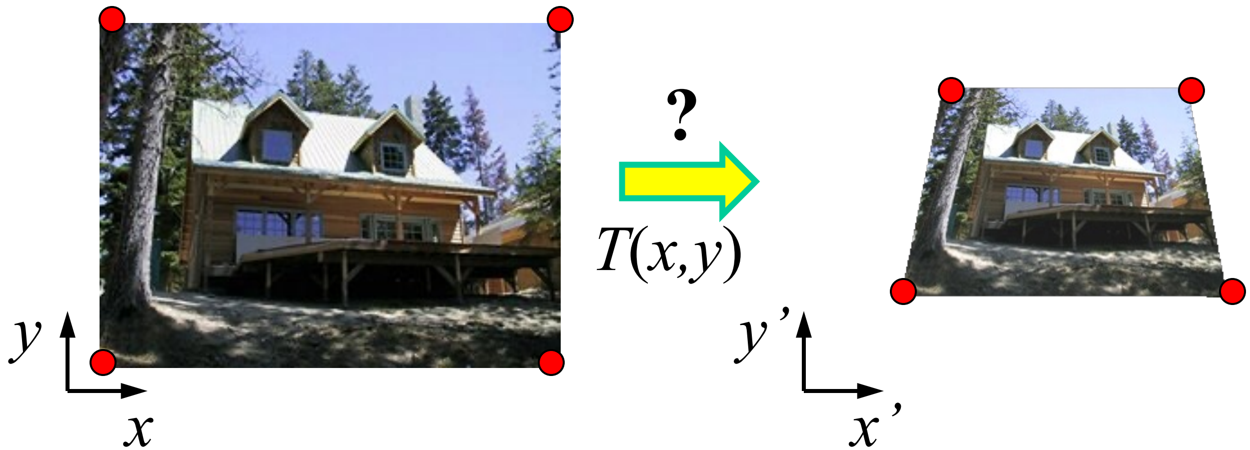
Affine: # correspondences?



How many correspondences needed for affine?

How many DOF?

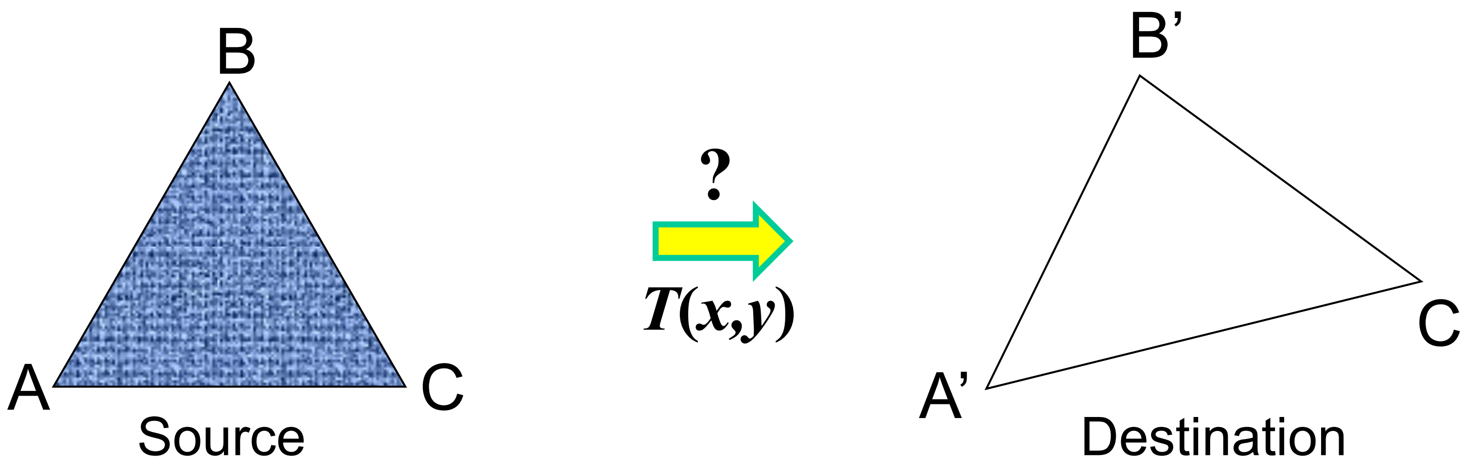
Projective: # correspondences?



How many correspondences needed for projective?

How many DOF?

Example: warping triangles



Given two triangles: ABC and A'B'C' in 2D (12 numbers)

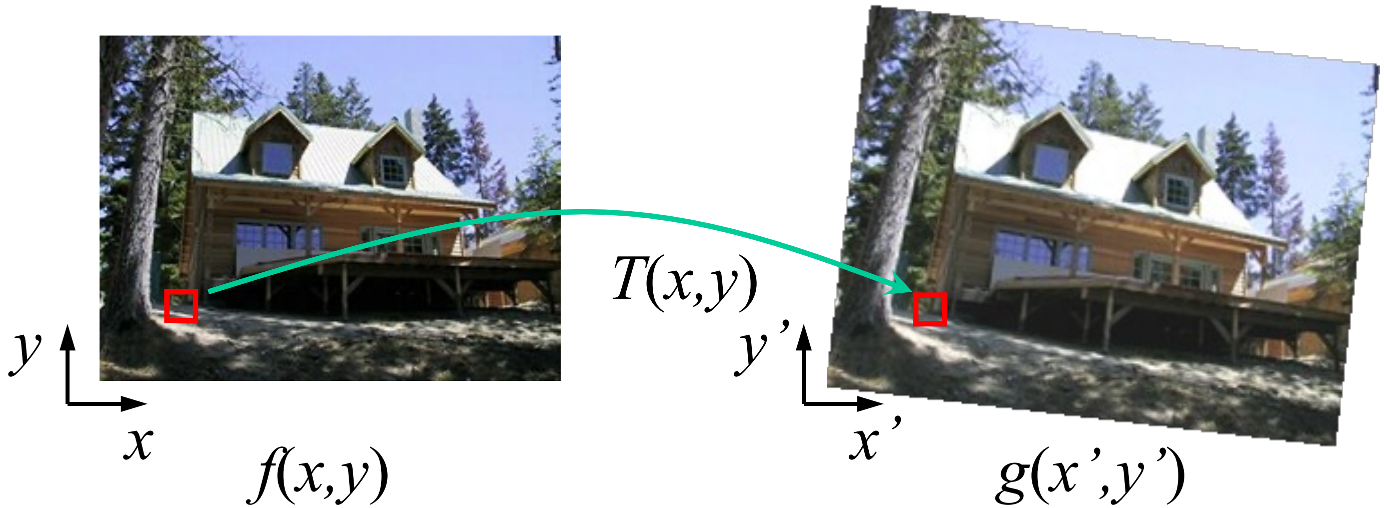
Need to find transform T to transfer all pixels from one to the other.

What kind of transformation is T?

How can we compute the transformation matrix:

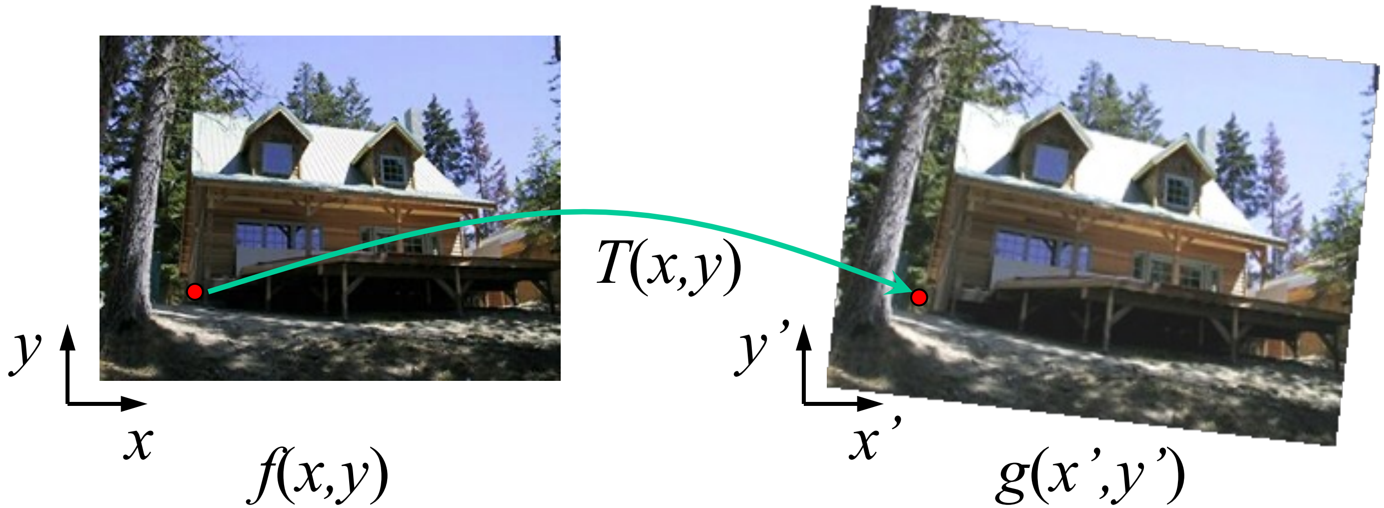
$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Image warping



Given a coordinate transform *(x',y')* = *T*(*x*,*y*) and a source image *f*(*x*,*y*), how do we compute a transformed image *g(x',y')* = *f*(*T*(*x*,*y*))?

Forward warping

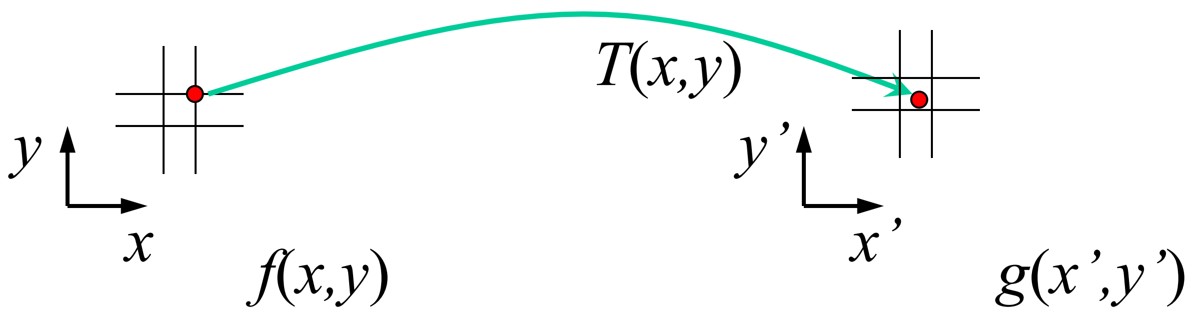


Send each pixel *f*(*x*,*y*) to its corresponding location

(x',y') = *T*(*x*,*y*) in the second image

Q: what if pixel lands "between" two pixels?

Forward warping



Send each pixel *f*(*x*,*y*) to its corresponding location

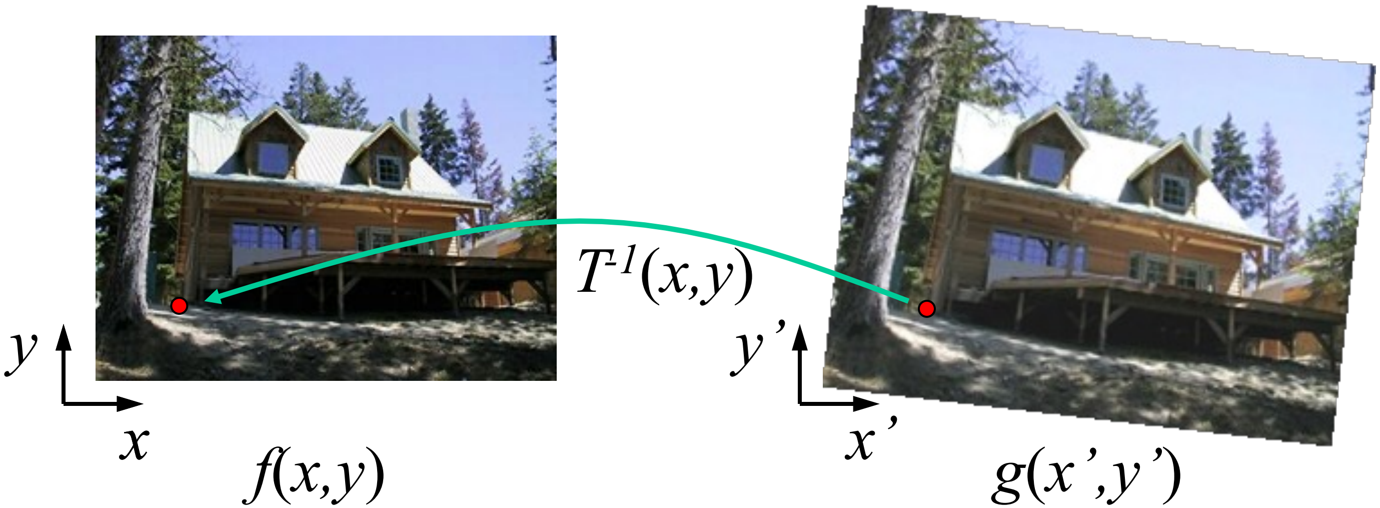
(*x'*,*y'*) = *T*(*x*,*y*) in the second image

Q: what if pixel lands "between" two pixels?

A: distribute color among neighboring pixels (x',y')

- Known as "splatting"
- Check out `griddata` in Matlab

Inverse warping

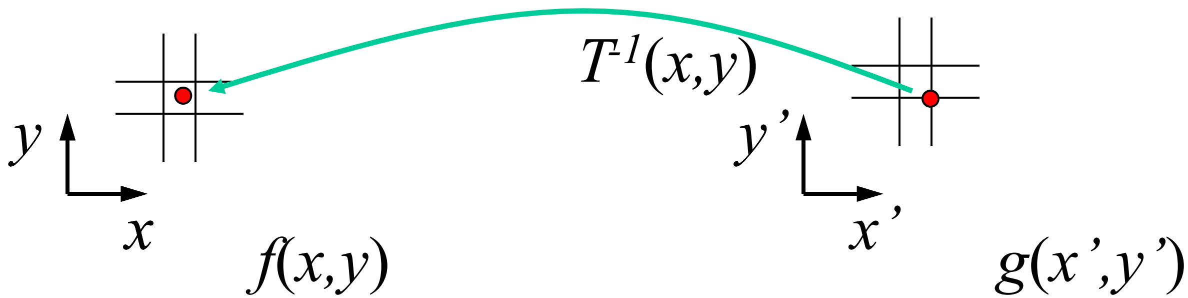


Get each pixel $g(x',y')$ from its corresponding location

$(x,y) = T^{-1}(x',y')$ in the first image

Q: what if pixel comes from “between” two pixels?

Inverse warping



Get each pixel $g(x',y')$ from its corresponding location

$$(x,y) = T^{-1}(x',y') \text{ in the first image}$$

Q: what if pixel comes from "between" two pixels?

A: *Interpolate* color value from neighbors

- nearest neighbor, bilinear, Gaussian, bicubic
- Check out `interp2` in Matlab

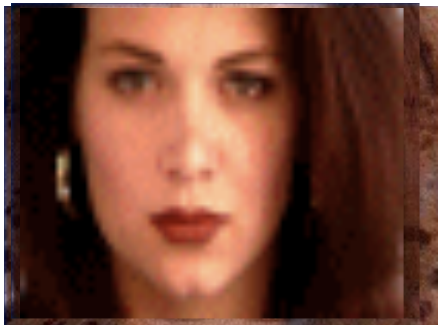
Forward vs. inverse warping

Q: which is better?

A: usually inverse—eliminates holes

- however, it requires an invertible warp function—not always possible...

Morphing = Object Averaging


The aim is to find "an average" between two objects

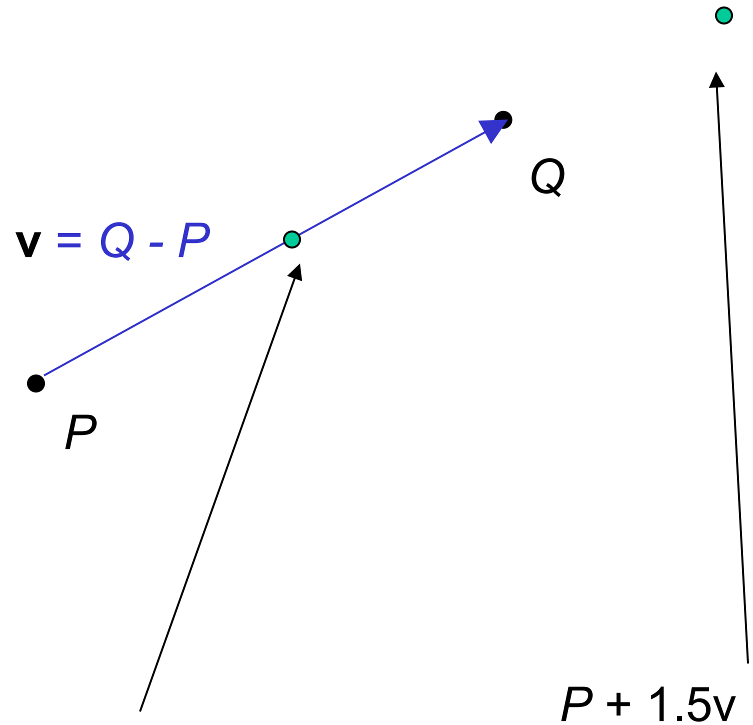
- Not an average of two images of objects…
- …but an image of the average object!
- How can we make a smooth transition in time?
  - Do a "weighted average" over time t

How do we know what the average object looks like?

- We haven't a clue!
- But we can often fake something reasonable
  - Usually required user/artist input

Averaging Points

What's the average of P and Q?



$P + 0.5v$
$= P + 0.5(Q - P)$
$= 0.5P + 0.5\,Q$

$P + 1.5v$
$= P + 1.5(Q - P)$
$= -0.5P + 1.5\,Q$
(extrapolation)

Linear Interpolation
(Affine Combination):
New point *aP* + *bQ*,
defined only when a+*b* = 1
So aP+bQ = aP+(1-a)Q

P and Q can be anything:

- points on a plane (2D) or in space (3D)
- Colors in RGB or HSV (3D)
- Whole images (m-by-n D)… etc.

Idea #1: Cross-Dissolve

Interpolate whole images:

$$Image_{halfway} = (1-t)*Image_1 + t*image_2$$

This is called **cross-dissolve** in film industry

But what is the images are not aligned?

Idea #2: Align, then cross-disolve

Align first, then cross-dissolve

- Alignment using global warp – picture still valid

Global warp not always enough!

What to do?

- Cross-dissolve doesn't work
- Global alignment doesn't work
  - Cannot be done with a global transformation (e.g. affine)
- Any ideas?

Feature matching!

- Nose to nose, tail to tail, etc.
- This is a local (non-parametric) warp

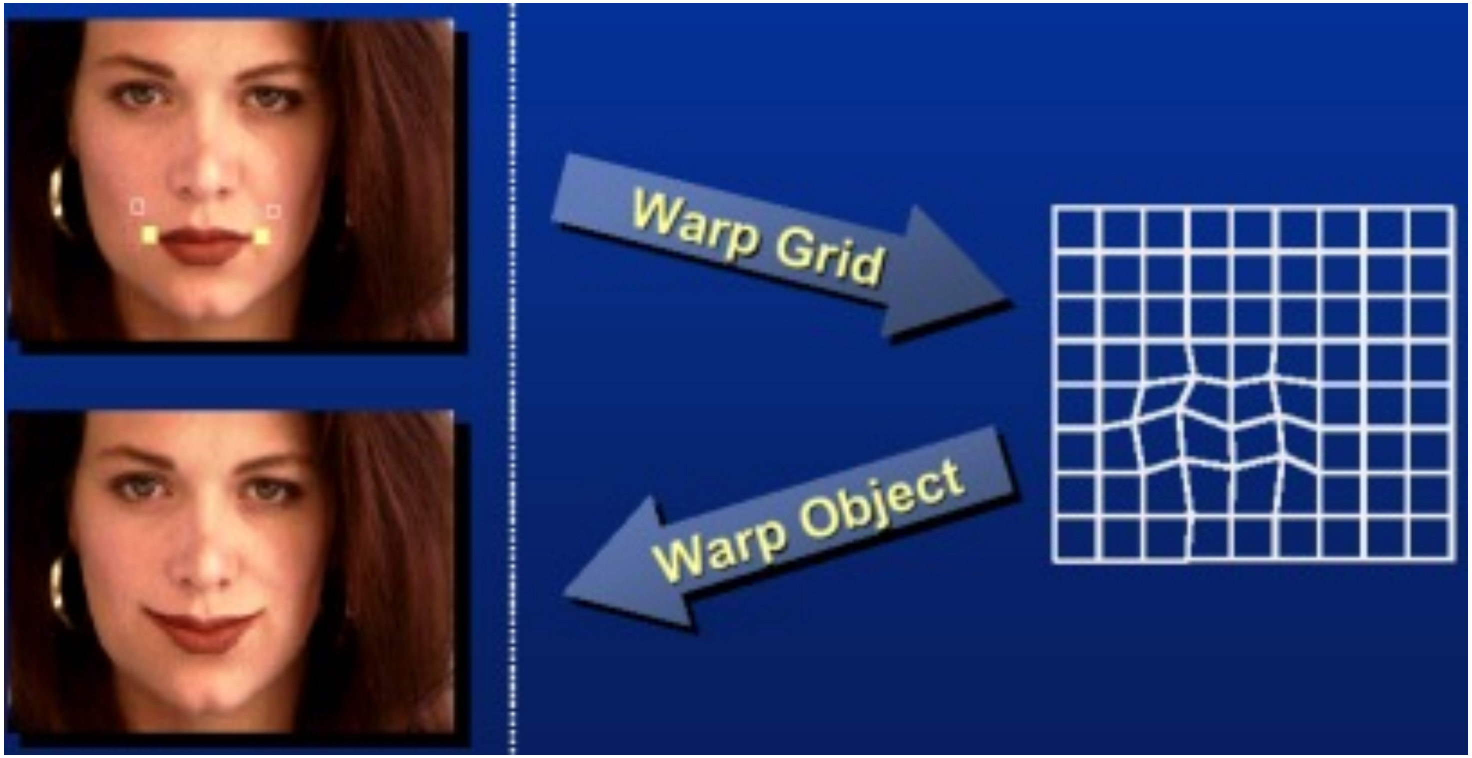
Local (non-parametric) Image Warping

Need to specify a more detailed warp function

- Global warps were functions of a few (2,4,8) parameters
- Non-parametric warps u(x,y) and v(x,y) can be defined independently for every single location x,y!
- Once we know vector field u,v we can easily warp each pixel (use backward warping with interpolation)

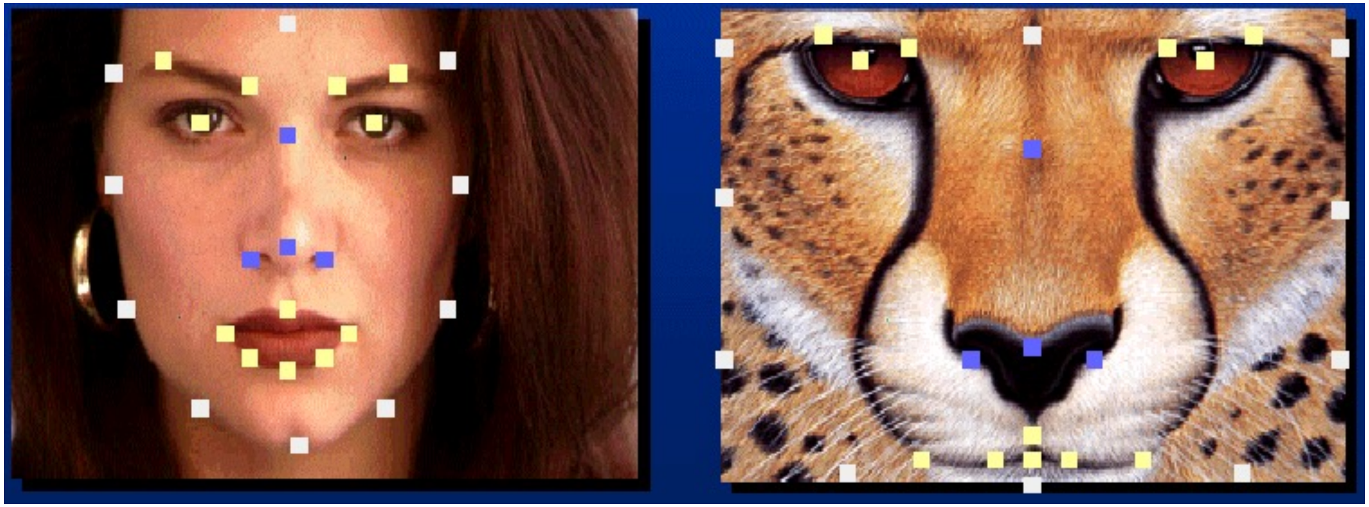
Warp specification -- dense

Define vector field to specify a dense warp



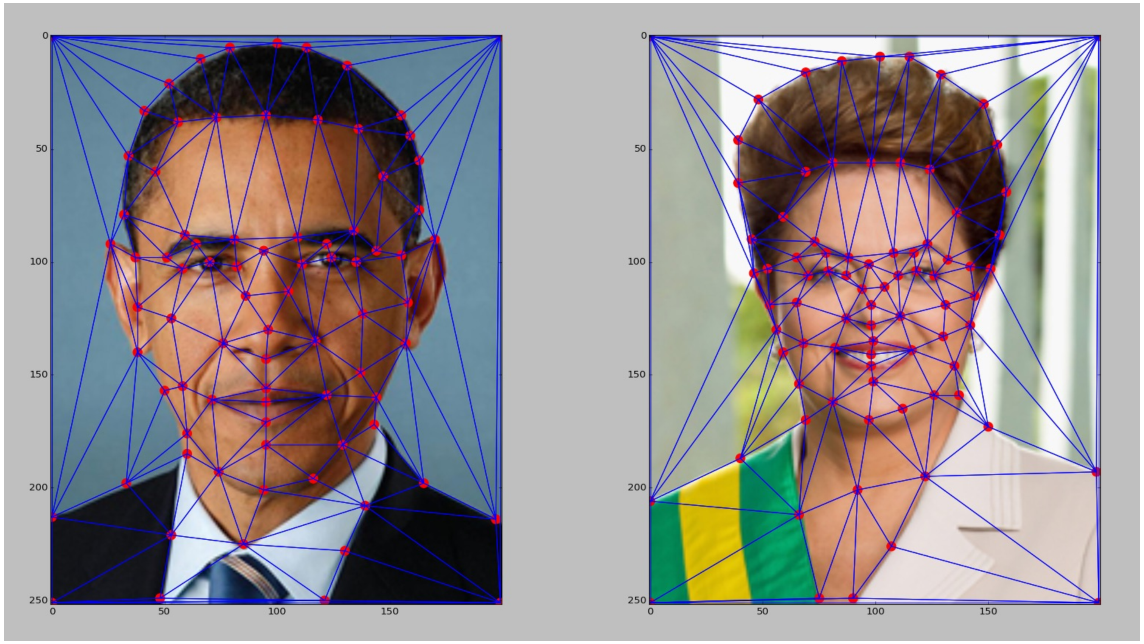
Warp specification - sparse

How can we specify a sparse warp?

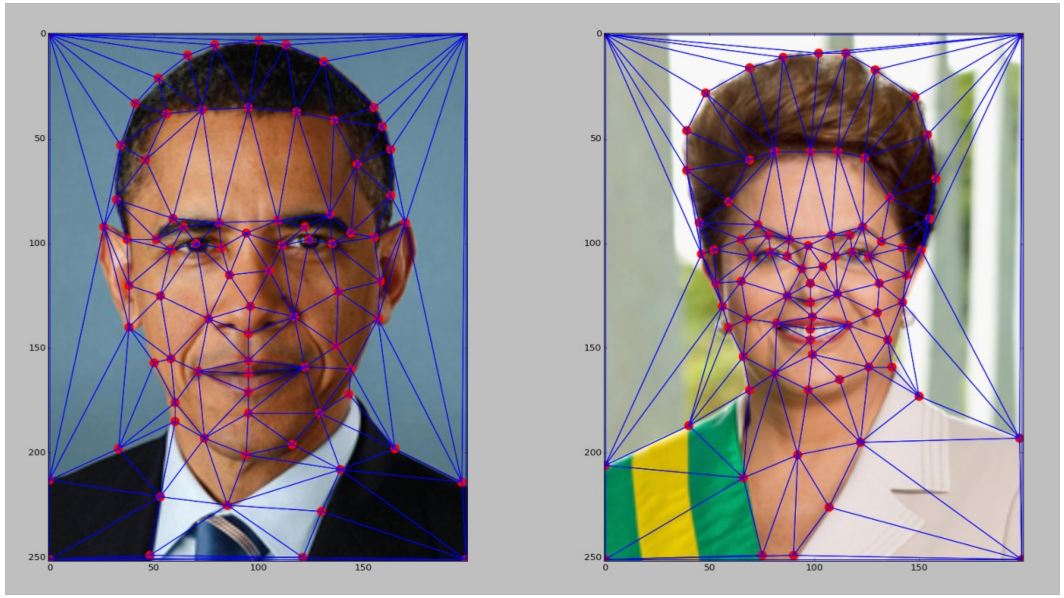


How do we go from feature points to pixels?

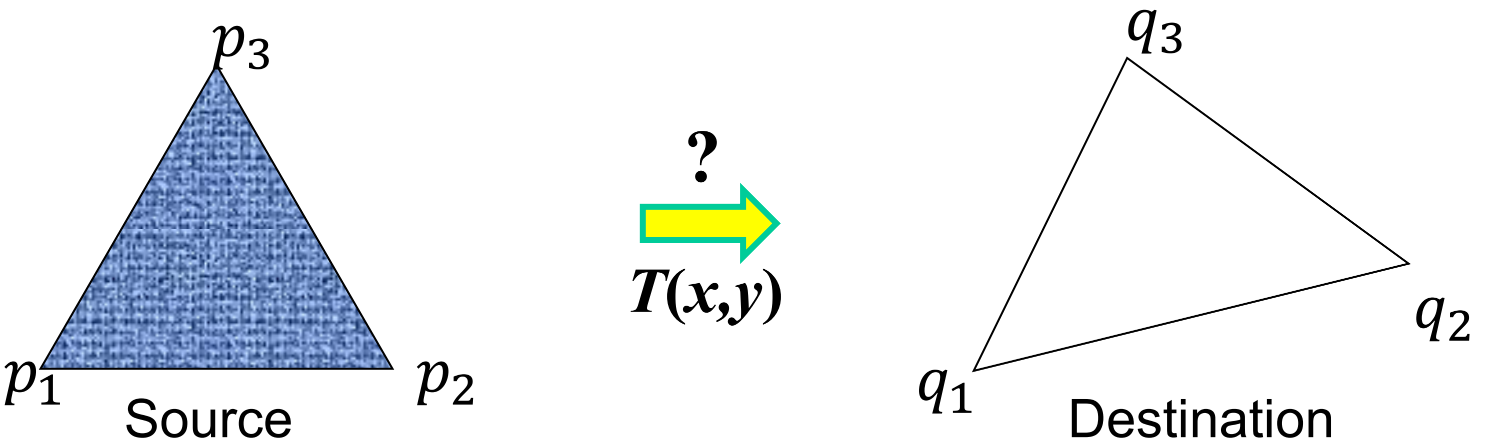
Triangular Mesh



1. Input correspondences at key feature points
2. Define a triangular mesh over the points
   - Same mesh in both images!
   - Now we have triangle-to-triangle correspondences
3. Warp each triangle separately from source to destination
   - How do we warp a triangle?


0
50
100
150
200
250
0
50
100
150

Warping triangles



Given two triangles: $p_1 p_2 p_3$ and $q_1 q_2 q_3$ in 2D (12 numbers)

Need to find transform T to transfer all pixels from one to the other.

What kind of transformation is T?

How can we compute the transformation matrix:

$\mathrm{p} = (x, y)$

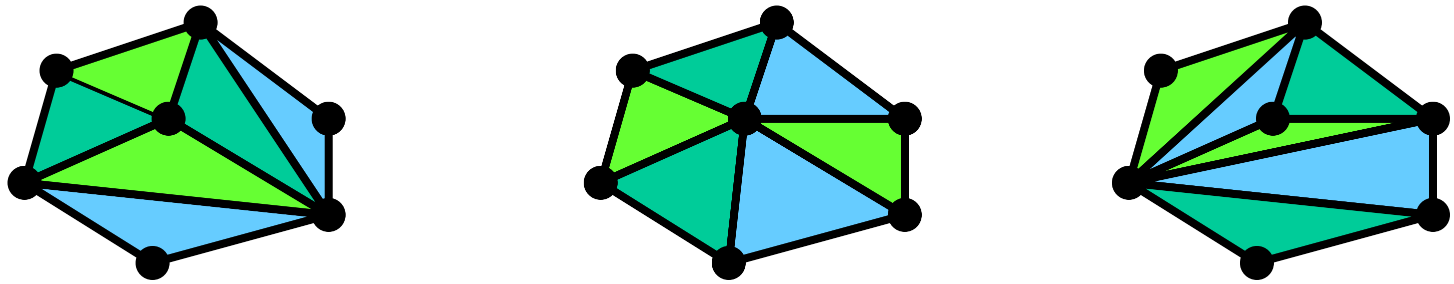
$q = (x', y')$

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

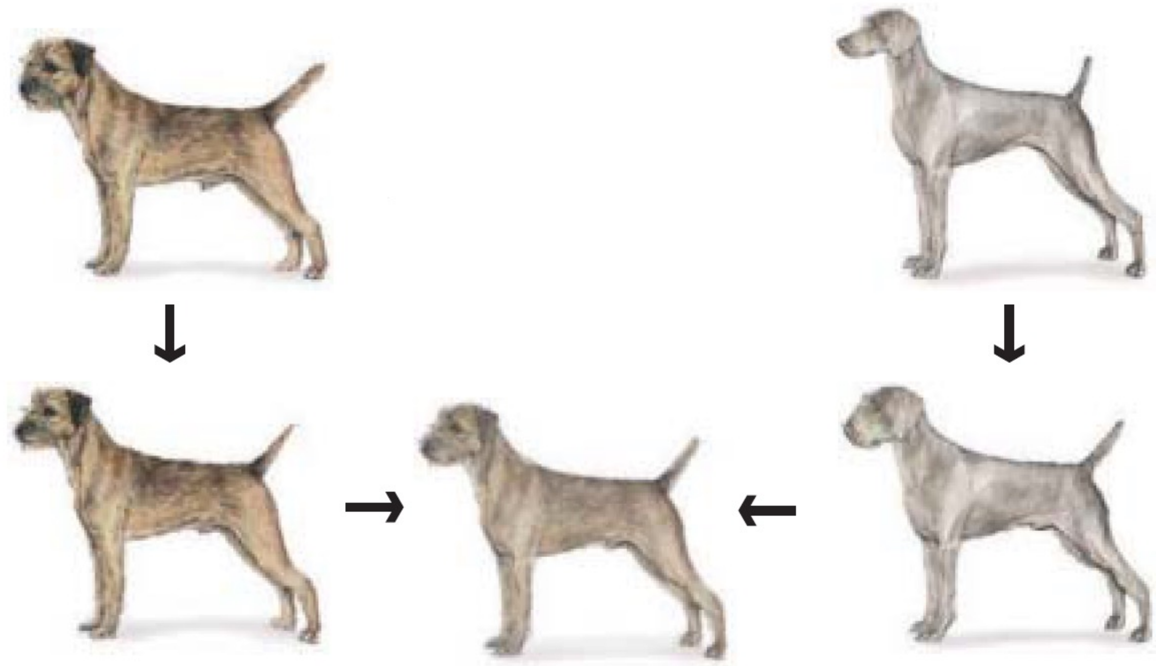
Triangulations

A *triangulation* of set of points in the plane is a *partition* of the convex hull to triangles whose vertices are the points, and do not contain other points.

There are an exponential number of triangulations of a point set.



Full Morphing Procedure



Morphing procedure:

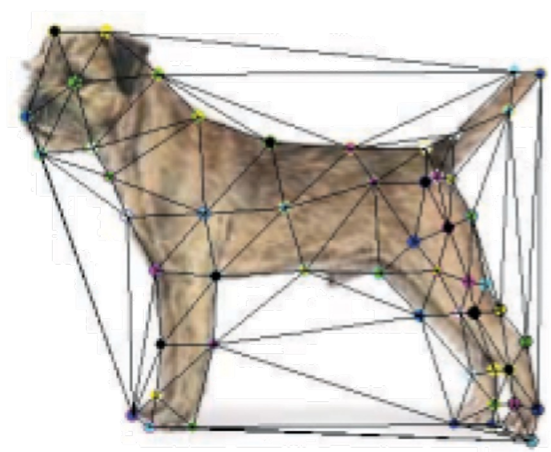
for every t,

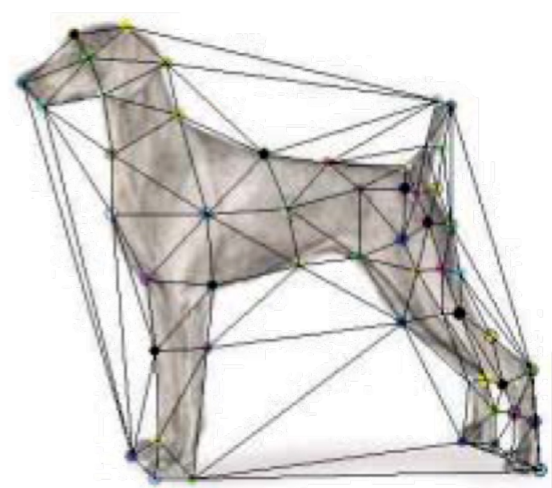
1. Find the average shape (the “mean dog”☺)
   - local warping
2. Find the average color
   - Cross-dissolve the warped images

1. Create Average Shape

How do we create an intermediate warp at time t?

- Assume t = [0,1]
- Simple linear interpolation of each feature pair
  - $p \rightarrow q$
  - $(1-t) \cdot p + t \cdot q$ for corresponding features p and p'





2. Create Average Color

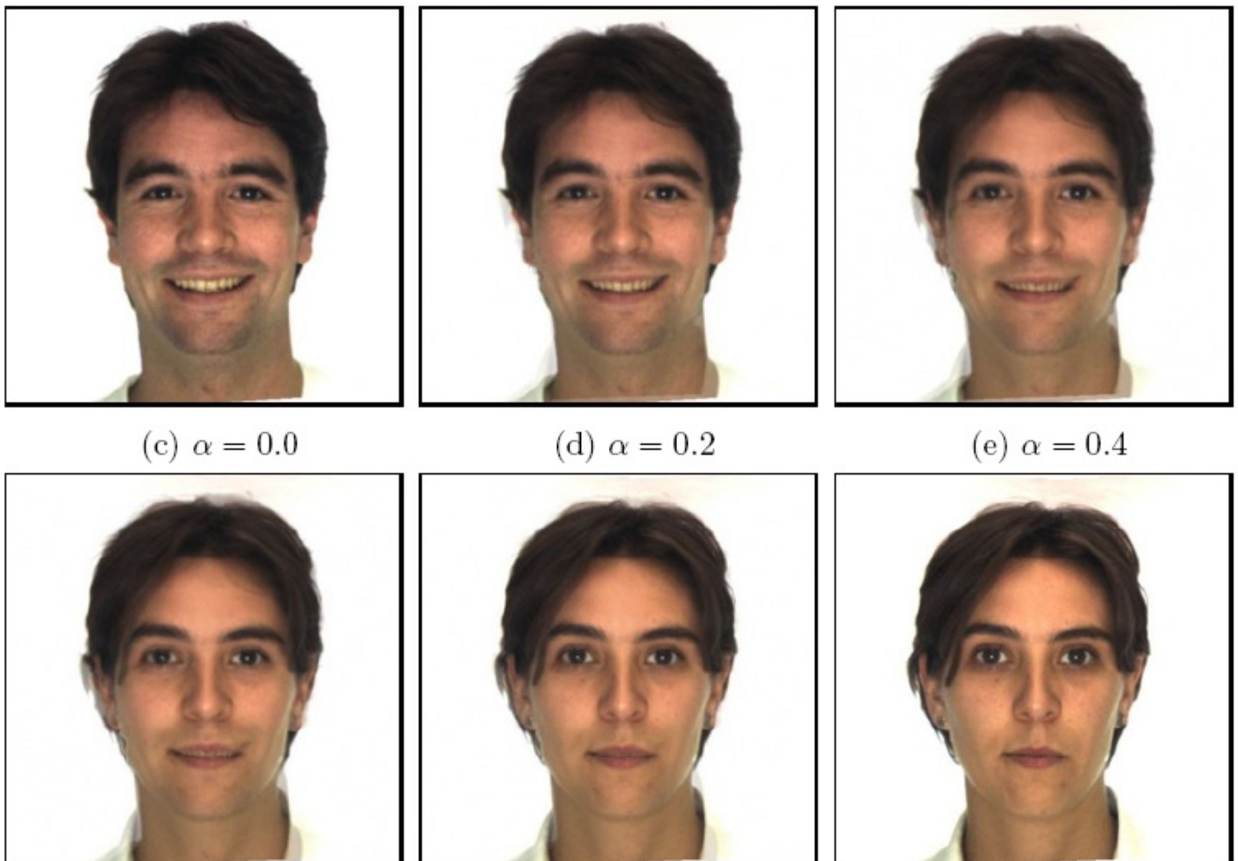
Interpolate whole images:

$\text{Image}_{\text{halfway}} = (1\text{-}t)^*\text{Image} + t^*\text{image}'$

cross-dissolve!

Morphing & matting

Extract foreground first to avoid artifacts in the background



(c) $\alpha = 0.0$ (d) $\alpha = 0.2$ (e) $\alpha = 0.4$

(f) $\alpha = 0.6$ (g) $\alpha = 0.8$ (h) $\alpha = 1.0$

Slide by Durand and Freeman

Amuse-bouche

By Philip Scott Johnson

Music: Bach's Sarabande from Suite for Solo Cello No. 1 in G Major, BWV 1007 performed by Yo-Yo Ma

Moving Least Square

What is a good local warping function $T(p) \rightarrow q$?

- Interpolation: need to satisfy control points $T(p_i) = q_i$
- Smoothness: T should be smooth;
- Identity: if $p_i = q_i$, T should be an identity mapping

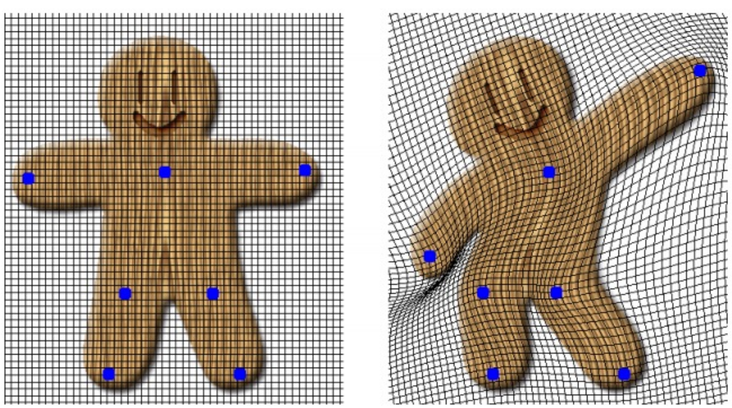
Triangulation-based methods:

$argmin_T \, ||T(p_i) - q_i||^2$ for 3 vertices in each triangle

Moving least squares:

$argmin_T \, w_i ||T(p_i) - q_i||^2$ for all the control points

Where $w_i = \frac{1}{|p_i - v|^{2\alpha}}$



v: current pixel

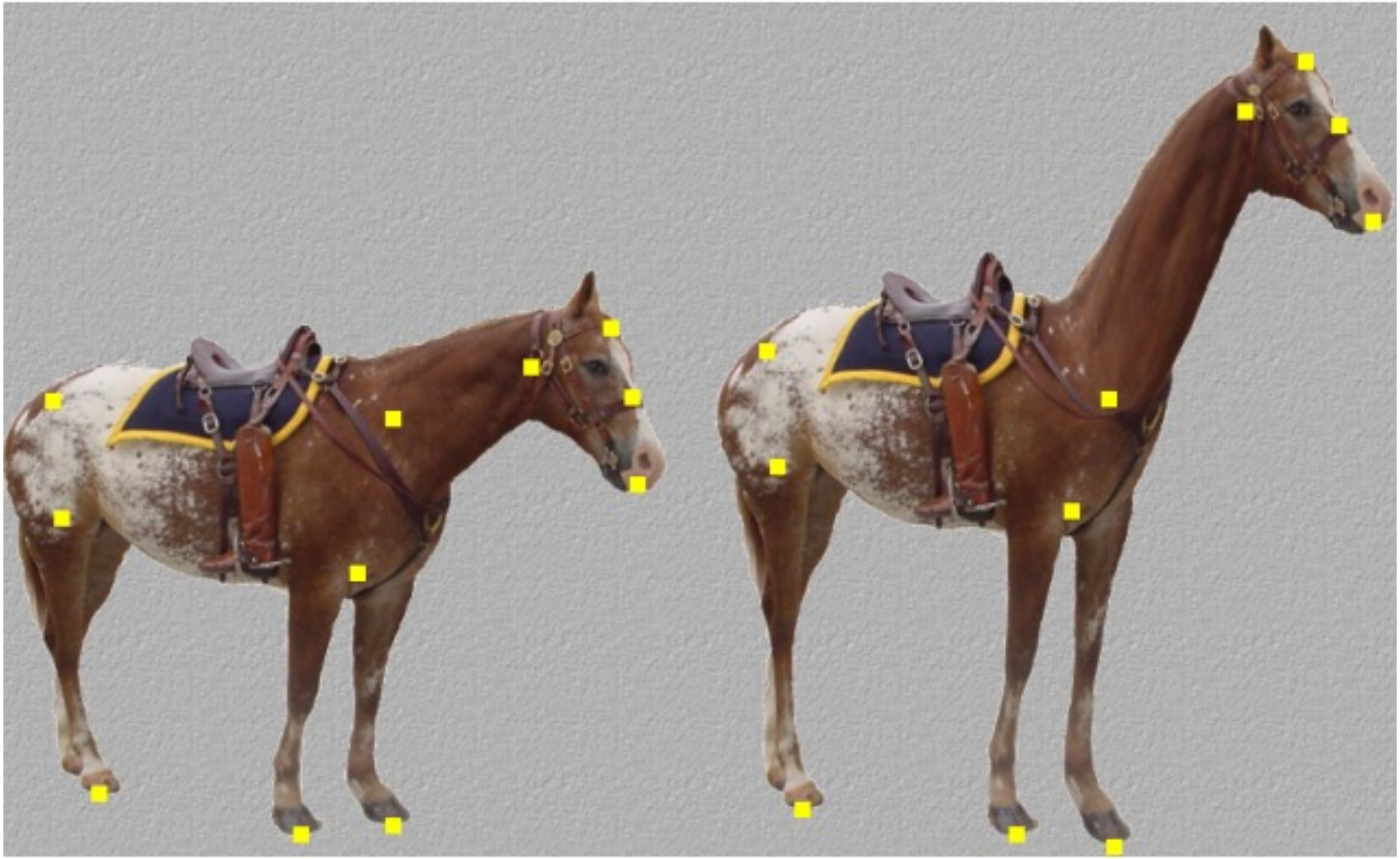
α: hyper-parameters

p_i: source control points

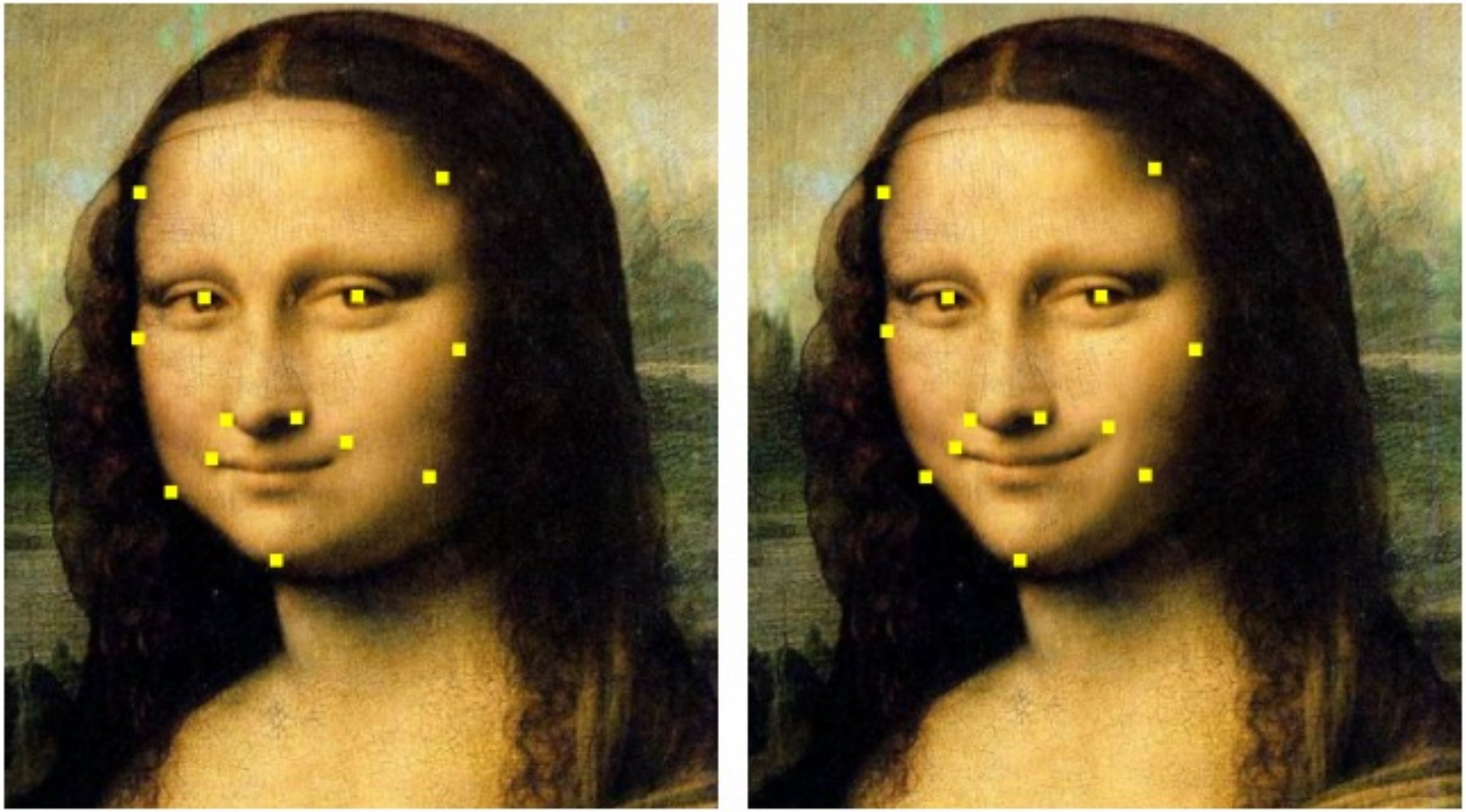
q_i: target control points

Image Deformation Using Moving Least Squares. Schaefer et al. SIGGRAPH 2006

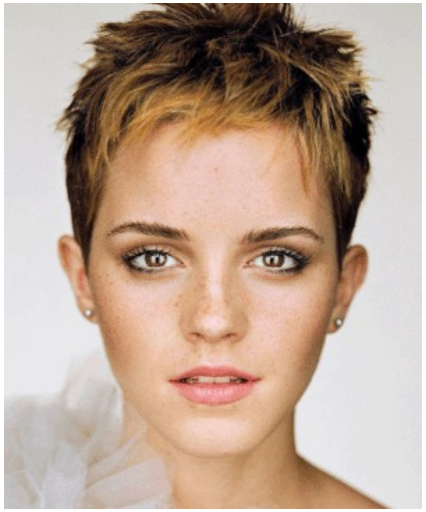
Moving Least Square



Moving Least Square



Thank You!

16-726, Spring 2023

https://learning-image-synthesis.github.io/